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the symbols in (5) occur on this triangle in the direction contrary to that of the hands of a watch, and the displacement of the letters takes place in this direction; thus

$$CRx = \begin{matrix} A & B \\ P & Q \end{matrix}.$$

On the other hand

$$RCx = \begin{matrix} B & P \\ A & Q \end{matrix}.$$

11. In comparing any two of the 24 square arrangements of the four letters, one of the letters may by the process indicated in equation (2) be brought into the same position in each form; the forms will then differ either in the position of two of the remaining letters indicating one of the relations denoted by R, C or J or else in the position of all three letters indicating one of the relations denoted by CR or RC, discussed above.

SOLUTION OF PROB. 407 BY PROF. EDGAR FRISBY.—I notice that the solution of problem 407 [see p. 158] contains a remarkable mistake in omitting the exponent $\frac{1}{2}$. The expression is an elliptic integral; it can be reduced to a series thus:

$$\int_0^1 \frac{(1+x^4)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}\,dx = \int_0^1 \frac{1+\frac{1}{2}x^4-\frac{1}{2}\cdot\frac{1}{4}x^8+\frac{1}{2}\cdot\frac{1}{4}\frac{3}{6}x^{12}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\frac{5}{8}x^{16}+-}{\sqrt{(1-x^2)}}\,dx.$$

Integrating each term by parts and only retaining the last term of each series, as the other terms vanish at both limits, we have

$$\int_0^{\frac{\pi}{2}} (1+\cos^4\theta)^{\frac{1}{2}}d\theta = \frac{\pi}{2}\left\{ \frac{1}{2}\frac{1.3}{2.4}-\frac{1.1}{2.4}\frac{1.3.57}{2.4.68}+\frac{1.1.3}{2.4.6}\frac{1.3.5.7.9.11}{2.4.6.8.10.12}\cdots \right\}.$$

SOLUTIONS OF PROBLEMS IN NUMBER FIVE.

SOLUTIONS of problems in No. 5 have been received as follows:

From Prof. L. G. Barbour, 412; Alex. S. Christie, 411; Prof, W. P. Casey, 409, 412, 416; Geo. Eastwood, 413, 416, 417; C. E. Everett, 409, 410, 415, 416; Prof. Edgar Frisby, 409, 411, 415; Wm. Hoover, 409, 411, 413 416; L. S. Hulbert, 412; E. H. Moore, Jr., 416; Prof. P. H. Philbrick, 409, 414, 415, 416; A. L. Parman, 409; Prof. E. B. Seitz, 409; Prof. J. Scheffer, 409, 410, 411, 416; Prof. C. M. Woodward, 409 410, 413; R. S. Woodward, 413.

[Solutions of 401, 402, 403 and 406 by Chas. E. Everett and Prof. Casey, respectively, were received too late for acknowledgment in No. 5.]

409. *By David Trowbridge, A. M., Waterburgh, N. Y.*—"If in any triangle ABC, squares be described on the three sides, and the vertices of the squares be joined by the three straight lines a, b, c; show that

$$a^2 + b^2 + c^2 = 3(AB^2 + BC^2 + CA^2)."$$

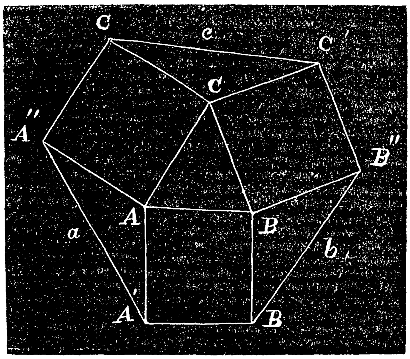
SOLUTION BY CHARLES E. EVERETT, SPIRIT LAKE, IOWA.

The angle $A'AA''$ is the supplement of A, $B'BB''$ is the supplement of B and $C'CC''$ is the supplement of C; hence

$$a^2 = AB^2 + AC^2 + 2AB.AC \cos A,$$
$$b^2 = AB^2 + BC^2 + 2AB.BC \cos B$$
$$c^2 = BC^2 + AC^2 + 2AC.BC \cos C.$$

Substituting for $\cos A$, $\cos B$ and $\cos C$ their values in terms of the sides of the triang and adding the eq's together,

$$a^2 + b^2 + c^2 = 3(AB^2 + AC^2 + BC^2).$$



410. *By Prof. J. Scheffer.*—"A cone with circular base is cut by a parabolic plane which passes through the centre of the base; to find the position of the centre of gravity of both portions of the cone."

SOLUTION BY C. M. WOODWARD, PH. D., WASH. UNIV., ST. LOUIS, MO.

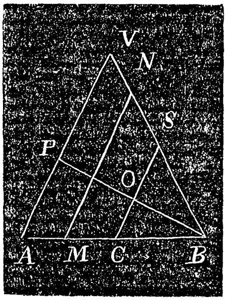
Let the plane of the figure be the meridian plane perpendicular to the given parabolic plane, and CS the trace of the given cutting plane. If $PV = \frac{3}{5}AV$, the straight line BP will pass through the center of gravity of the cone at O (giving $OV = \frac{3}{4}CV$) and through the centers required, since it passes through the centers of gravity of all the parllel parabolic laminæ.

Let MN be the trace of any one of these laminæ; its distance from B, measured on BP, be z; the height of the lamina x; and its limiting ordinate y.

From the figure $x = sz \div l$ (in which s is the slant height of the cone, and l is the length BP), and $y^2 = (zl - z^2)(4r^2 \div l^2)$. Hence the area of the lamina is

$$\tfrac{4}{3}xy = \frac{8}{3}\cdot\frac{rs}{l^2}\cdot\sqrt{(z^3l - z^4)}.$$

Finding the moment of the lamina about an axis at B,



substituting in formula for center of gravity, and cancelling constants, we have the general expression

$$z_0 = \frac{\int (zl-z^2)^{\frac{1}{2}} z^2 dz}{\int (zl-z^2)^{\frac{1}{2}} z\, dz},$$

in which the proper limits are to be inserted. The general value of the first integral is

$$-\tfrac{1}{4}z^{\frac{5}{2}}(l-z)^{\frac{3}{2}}-\tfrac{5}{24}lz^{\frac{3}{2}}(l-z)^{\frac{3}{2}}-\tfrac{5}{32}l^2z^{\frac{1}{2}}(l-z)^{\frac{3}{2}}+\tfrac{5}{64}l^3z^{\frac{1}{2}}(l-z)^{\frac{1}{2}}$$
$$+\tfrac{5}{128}l^4 \text{ arc ver. sin } (2z\div l)$$

The general value of the second integral is

$$-\tfrac{1}{3}z^{\frac{3}{2}}(l-z)^{\frac{3}{2}}-\tfrac{1}{4}lz^{\frac{1}{2}}(l-z)^{\frac{8}{2}}+\tfrac{1}{8}l^2z^{\frac{1}{2}}(l-z)^{\frac{1}{2}}+\tfrac{1}{16}l^3 \text{ arc ver. sin } (2z\div l).$$

For the smaller of the two cone segments the limits are 0 and $\frac{1}{2}l$; for the other $\frac{1}{2}l$ and l. Putting in the limits we have

$$z_1 = l\frac{\frac{5}{256}\pi-\frac{5}{192}-\frac{1}{64}}{\frac{1}{32}\pi-\frac{1}{24}} = \tfrac{5}{8}l-\tfrac{3}{2}\cdot\frac{l}{3\pi-4},$$

$$z_2 = l\frac{\frac{5}{128}\pi-\frac{5}{256}\pi+\frac{5}{192}+\frac{1}{16}}{\frac{1}{16}\pi-\frac{1}{32}\pi+\frac{1}{24}} = \tfrac{5}{8}l+\tfrac{3}{2}\cdot\frac{l}{3\pi+4};$$

or in general

$$z_0 = \tfrac{5}{8}l\pm\tfrac{3}{2}.l\div(3\pi\pm4),$$

in which the upper signs refer to the greater, and the lower signs to the smaller segment.

The first term in this result is the distance from *B* to *O*, the center of gravity of the entire cone, as is also evident if we take limits 0 and l.

The ratio of the distances of the two centers from the center of the cone is $3\pi-4 : 3\pi+4$, hence that is the inverse ratio of their volumes.

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411. *By Alex. S. Christie, U. S. Coast Survey.*—"Sum the series

$$1-\frac{n}{1}\frac{1}{3}+\frac{n(n-1)}{2!}\frac{1}{5}-\frac{n(n-1)(n-2)}{3!}\frac{1}{7}+ \&c.,$$

for positive values of n."

SOLUTION BY THE PROPOSER.

$$\text{Let } \Sigma = 1-\frac{n}{1}\frac{1}{3}+\frac{n(n-1)}{2!}\frac{1}{5}-\frac{n(n-1)(n-2)}{3!}\frac{1}{7}+\&\text{c., and}$$

$$S = x-\frac{n}{1}\frac{x^3}{3}+\frac{n(n-1)}{2!}\frac{x^5}{5}-\frac{n(n-1)(n-2)}{3!}\frac{x^7}{7}+\&\text{c.}$$

$$\text{Then } \frac{dS}{dx} = 1-\frac{n}{1}x^2+\frac{n(n-1)}{2!}x^4-\frac{n(n-1)(n-2)}{3!}x^6+\&\text{c.} = (1-x^2)^n$$

$$\text{and} \quad S = \int_0^x (1-x^2)^n dx = \tfrac{1}{2}\int_{-x}^{+x}(1-x^2)dx;$$

$$\therefore\ \Sigma = \tfrac{1}{2}\int_{-1}^{+1}(1-x^2)^n dx = \tfrac{1}{2}\int_{+1}^{+1}(1+x)^n(1-x)^n dx.$$

Put $1+x = 2y$, $\therefore 1-x = 2(1-y)$, $dx = 2dy$.

Then $\quad \Sigma = 2^{2n}\int_0^1 y^n(1-y)^n dy = 2^{2n}B(n+1,\, n+1)$

$$= 2^{2n}.\frac{\Gamma(n+1)\,\Gamma(n+1)}{\Gamma(2n+2)}.$$

SOLUTION BY WM. HOOVER, A. M., DAYTON, OHIO.

The given series is the value of

$$\int_0^1(1-x^2)^n dx = \frac{2n\,.\,2(n-1)\;2(n-2)\;.\,.\,.\,.\;2}{(2n+1)(2n-1)(2n-3)\dots n},$$

which is therefore the required sum.

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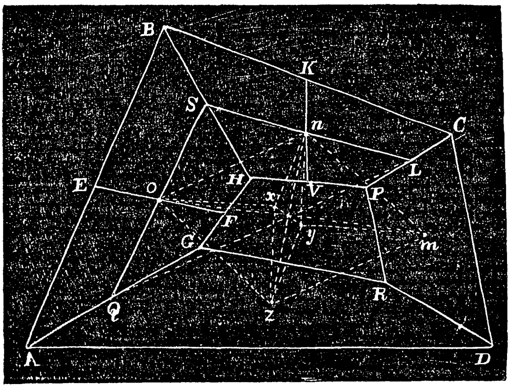
412. *By Prof. L. G. Barbour.*—"Show that in any hexaedron bounded by quadrilaterals, the three lines respectively connecting the mean points of opposite (non-contiguous) faces, mutually bisect each other."

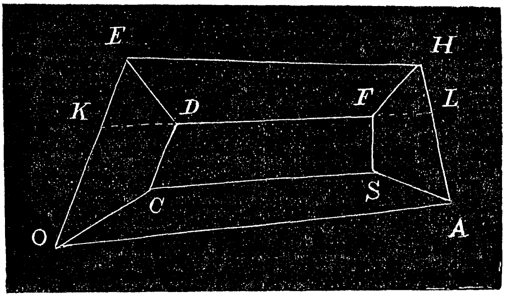
SOLUTION BY PROF. W. P. CASEY, SAN FRANCISCO, CAL.



Let *ABCDRPHG* be the hexaedron, and *Q*, *E*, *S*, *F*, *K* *L*, *V* the middle points of *AG* *AB*, *BH*, *HF*, *BC*, *CP*, *PH* Join *QS* and *EF*, *SL* and *KV* intersecting in *o* and *n*, which are the mean points of the faces *AH*, *HC*, and as $So = oQ$ and $Sn = nL$, $\therefore$ *on* is parallel to *QL* and equal to one-half of it And if *m*, *z* be the mean points of the faces *AR*, *RC*, *zm* is for a like reason parallel to *QL* and equal to one-half of it, and so is *mn* equal and parallel to *oz*, *xz* to *yn*, *xn* to *yz* and *ox* to *my* (*x*, *y* being the mean p'ts of the faces *AC*, *GP*); $\therefore$ *xy*, *om*, *nz* are concurrent and bisect each other.

[Prof. Barbour and Mr. Hulbert, each, demonstrates this proposition by quaternions, and Prof. Barbour appends to his demonstration the following corollaries.]

Cor. 1. The face *BCDF* may approach and finally coincide with the plane of *AOEH*, and the volume degenerate into a plane surface as in the Fig. Thus, the lines joining the mean points of *EDFH* and *AOCB*



OCDE and *ABFH, BCDF* and *AOEH* will mutually bisect each other.

Cor. 2. If *E* should app'ch until it reached *K* where *EO* is intersected by *FD* produced; and *H* app'ch *A* until it reaches *L* where *HA* is intersected by *DF* produced, the theorem would still hold good with this modification; viz., the mean point of *EDFH* would become the middle point of *KL, EOCD* would become *KOCD*, and *HFBA, LFBA*.

Cor. 3. Continuing the process, we should at length have left only *BCDF*. The lines joining the middle points of the opposite (non-contiguons) sides would bisect each other in the mean point, which is thus a special case of the general theorem.

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413. *By William Hoover, A. M.*—"A rod rests with one extremity in a smooth plane and the other against a smooth vertical wall at an inclination α to the horizon. If it then slips down, show that it will leave the wall when its inclination is $\sin^{-1}(\frac{2}{3}\sin\alpha)$."

SOLUTION BY R. S. WOODWARD, C. E., DETROIT, MICH.

Let the mass of the rod be m, its length $2l$, its inclination to the horizon θ and the coordinates of its centre of gravity x, y, the origin being such that for the time, t, considered, $x = l\cos\theta$ and $y = l\sin\theta$. Let the horizontal and vertical reactions at the ends of the rod be H and V respectively. Then the equations of motion are

$$m\frac{d^2y}{dt^2} = ml\frac{d^2(\sin\theta)}{dt^2} = -my + V, \qquad (1)$$

$$m\frac{d^2x}{dt^2} = ml\frac{d^2(\cos\theta)}{dt^2} = +H, \qquad (2)$$

$$\tfrac{1}{3}ml^2\frac{d^2\theta}{dt^2} = Hl\sin\theta - Vl\cos\theta. \qquad (3)$$

Multiply (1) by $\cos\theta$, (2) by $-\sin\theta$ and (3) by $(1\div l)$ and add the products. There results

$$\tfrac{4}{3}ml\frac{d^2\theta}{dt^2} = -mg\cos\theta, \qquad (4)$$

whence, since $d\theta\div dt = 0$ and $\theta = \alpha$ when $t = 0$,

$$\frac{d\theta^2}{dt^2} = \frac{3}{2}\frac{g}{l}(\sin\alpha - \sin\theta). \qquad (5)$$

The rod will leave the vertical wall when

$$H = -mt\left(\sin\theta\frac{d^2\theta}{dt^2} + \cos\theta\frac{d\theta^2}{dt^2}\right) = 0.$$

Substituting in this the values of $d^2\theta \div dt^2$ and $d\theta^2 \div dt^2$ given by (4) and (5) there results

$$\theta = \sin^{-1}(\tfrac{2}{3}\sin \alpha).$$

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414. "Sum the series, $\sec\theta + \sec\tfrac{1}{2}\theta + \sec\tfrac{1}{4}\theta + \sec\tfrac{1}{8}\theta + \ldots + \sec\tfrac{1}{2^n}\theta$."

SOLUTION BY PROF. P. H. PHILBRICK.

$$\text{Sec}\,\theta = 1 + \tfrac{1}{2}\theta^2 + \tfrac{5}{24}\theta^4 + \ldots\ldots = 1 + a\theta^2 + b\theta^4 + \ldots\ldots \text{ say;}$$

$$\therefore \sec\tfrac{1}{2}\theta = 1 + (\tfrac{1}{2})^2 a\theta^2 + (\tfrac{1}{2})^4 b\theta^4 + \ldots\ldots$$

$$\sec\tfrac{1}{4}\theta = 1 + (\tfrac{1}{4})^2 a\theta^2 + (\tfrac{1}{4})^4 b\theta^4 + \ldots\ldots$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$\sec(\tfrac{1}{2})^n\theta = 1 + (\tfrac{1}{2})^{2n} a\theta^2 + (\tfrac{1}{2})^{4n} b\theta^4 + \ldots\ldots$$

Now the corresponding terms of these partial series form geometrical series; hence,

$$\sec\theta + \sec\tfrac{1}{2}\theta + \sec\tfrac{1}{4}\theta + \ldots + \sec(\tfrac{1}{2})^n\theta \text{ (to } n+1 \text{ terms)}$$

$$= n+1+a\theta^2[1+\tfrac{1}{4}+\tfrac{1}{16}+\ldots\ldots+(\tfrac{1}{4})^n]$$

$$+b\theta^4[1+\tfrac{1}{16}+\tfrac{1}{256}+\ldots\ldots+(\tfrac{1}{16})^n]$$

$$= n+1+a\theta^2\left[\frac{1-(\frac{1}{4})^{n+1}}{1-\frac{1}{4}}\right] + b\theta^4\left[\frac{1+(\frac{1}{16})^{n+1}}{1-\frac{1}{16}}\right] + \ldots$$

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415. "Evaluate $\displaystyle\int\frac{dx}{x-dx}$."

SOLUTION BY PROF. FRISBY.

$$\int_0^x \frac{x}{x-dx} = \int\frac{dx}{x}\left(1-\frac{dx}{x}\right)^{-1} = \int\frac{dx}{x} + \left(\frac{dx}{x}\right)^2 + \left(\frac{dx}{x}\right)^3 + \text{\&c.},$$

$$= \log x;$$

all terms after the first being evanescent.

[Prof. Philbrick and Mr. Everett obtain the same result as above. The problem was inserted for the benefit of a correspondent who insists that Prof. Bartlett is in error for treating a similar expression (see *Acoustics and Optics*, p. 29, line 3; edition of 1853) in accordance with the above solution and whose criticism was declined because we believed it to be erroneous, but who, nevertheless, insists on our presenting the subject to the readers of the ANALYLT.—Ed.

416. *By Prof. W. P. Casey.*—"Given the base AB and the angle A of a triangle ABC; find the locus of the foot of the perpendicular CF drawn from C to the side of the inscribed square."

SOLUTION BY GEO. EASTWOOD.

Let $GHIJ$ denote the inscribed square, and let x, y be the coordinates of the point F, A being the origin. Put $AB = a$ and the tangent of the given angle $= t$; then is (K denoting the intersect. of CF produced with AB) $CK = tx$, $CF = tx—y$; and $\triangle ABC : \triangle GHC :: AB^2 : GH^2 (= FK^2)$, which gives $tx : tx—y :: a : y$, or $txy + ay — atx = 0$, which being of the form $Bxy + Dy + Ex = 0$, dentes a hyperbola whose asymptotes are represented by the equations $Bx + D = 0$, $By + E = 0$.

[For want of room, the solution of 417 is deferred to a future number.]

PROBLEMS.

418. *By Levi W. Meech, A. M., Norwich, Conn.*—Required to express Lagrange's Theorem in terms of Finite Differences, as far as practicable, instead of the usual differentials.

419. *By C. E. Everett, Spirit Lake, Iowa.*—Find the locus of a point starting from the centre of a given circle and moving so that the arc included between any two postions of the point shall equal the arc of the circle intercepted by the radii drawn through the same positions.

420. *By Prof. Asaph Hall.*—Transform the definite integral

$$\int_b^a \varphi(x).dx,$$

so that the limits of integration shall be m and n.

421. *By George Eastwood, Saxonville, Mass.*—In a Bicycle exercise on a level, circular course of given radius, what angle ought the plane of the machine to make with the vertical, so that the rider may move on the circumference of a perfect circle?

421. *By W. E. Heal, Marion, Indiana.*—Determine the most general form of two algebraic functions φ and θ such that

$$\varphi(x) + \varphi(y) = \varphi[\theta(x, y)],$$

or prove that there are no such functions.